

# Creation of an inflationary universe out of a black hole

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We investigate the effect of a black hole as a nucleation cite of a false vacuum bubble based on the Euclidean actions of relevant configurations. As a result we find a wormhole-like configuration may be spontaneously nucleated once the black hole mass falls below a critical value of order of the Hubble parameter corresponding to the false vacuum energy density. As the space beyond the wormhole throat can expand exponentially, this may be interpreted as creation of another inflationary universe in the final stage of the black hole evaporation.

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Inflation in the early universe provides answers to a number of fundamental questions in cosmology such as why our Universe is big, old, full of structures, and devoid of unwanted relics predicted by particle physics models [1]. Furthermore, despite the great advancements in precision observations of cosmic microwave background (CMB) radiation, there is no observational result that is in contradiction with inflationary cosmology so far [2, 3].

Inflationary cosmology has also revolutionized our view of the cosmos, namely, our Universe may not be the one and the only entity but there may be many universes. Indeed already in the context of the old inflation model [4, 5], Sato and his collaborators found possible production of child (and grand child...) universes [6–8].

Furthermore, if the observed dark energy consists of a cosmological constant  $\Lambda$ , our Universe will asymptotically approach the de Sitter space which may up-tunnel to another de Sitter universe with a larger vacuum energy density [9–11] to induce inflation again to repeat the entire evolution of another inflationary universe. In such a recycling universe scenario, the Universe we live in may not be of first generation, and we may not need the real beginning of the cosmos from the initial singularity [12].

In this context, so far the phase transition between two pure de Sitter space has only been considered. However, phase transitions which we encounter in daily life or laboratories are usually induced around some impurities which act as catalysts or boiling stones. In cosmological phase transitions, black holes may play such roles. This issue was pioneered by Hiscock [13]. More recently, Gregory, Moss and Withers revisited the problem [14]. They have observed that the black hole mass may change in the phase transition and calculated the Euclidean action taking conical deficits into account [14–16].

In this manuscript we report the effect of a black hole on up-tunneling assuming that the high energy theory of elementary interactions accommodate a false vacuum with energy density  $U = M_X^4 \equiv 3M_{Pl}^2 H^2$ , where  $M_{Pl}$  is the reduced Planck scale, and that transition be-

tween this state and the current vacuum state is possible through a thin wall bubble nucleation with its surface tension  $\sigma$  (See Fig. 1). We assume the energy scale  $M_X$  is somewhat smaller than the typical grand unification scale  $M_{GUT} \sim 10^{16}\text{GeV}$  on the basis of the constraints imposed on the energy scale of inflation from B-mode polarization of CMB [17]. As a result we show that as the black hole mass decreases to  $\sim M_{Pl}^3/M_X^2$  due to the Hawking radiation [18, 19], a false vacuum bubble may be spontaneously nucleated, to create a wormhole-like configuration. Beyond the throat is a false vacuum state

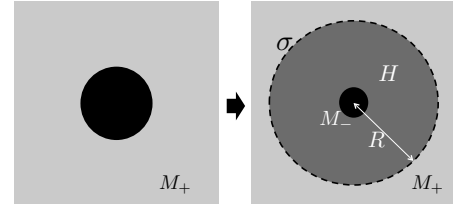


FIG. 1: Phase transition from an original black hole with a mass  $M_+$  to a false vacuum bubble with the energy density  $U = 3M_{Pl}^2 H^2$  separated by a bubble wall with its surface tension  $\sigma$  and a remnant black hole with a mass  $M_-$ .

which inflate to create another big universe. Then one may regard that the final fate of an evaporating black hole is actually another universe.

We study how such a configuration may be created from an initially Schwarzschild geometry with the mass parameter  $M_+$  by calculating Euclidean actions of initial and final configurations. Since the energy scale of the false vacuum is presumably much larger than that of the current dark energy, we neglect the latter. To be more specific, we consider the case a false vacuum bubble is nucleated around aforementioned Schwarzschild black hole and its radius  $R$  expands to create a big inflationary domain, leaving a black hole with mass  $M_-$  in the center which may be different from  $M_+$ .

After the bubble nucleation, the inner geometry labeled with a suffix  $-$  is Schwarzschild de Sitter space, which is connected with the outer Schwarzschild geometry labeled by a suffix  $+$  by a thin wall bubble with surface tension  $\sigma$ . Since such a local process cannot change the outer geometry, it must remain Schwarzschild space-time with mass  $M_+$ . Then the inner and outer metrics are given by

$$ds^2 = -f_{\pm}(r)dt^2 + \frac{dr^2}{f_{\pm}(r)} + r^2 d\Omega^2, \quad (1)$$

$$f_+(r) \equiv 1 - \frac{2GM_+}{r}, \quad f_-(r) \equiv 1 - \frac{2GM_-}{r} - H^2 r^2.$$

We describe the wall trajectory in terms of the local coordinates  $(t_{\pm}(\tau), r_{\pm}(\tau), \theta, \varphi)$  on each side depending on the proper time  $\tau$  of an observer on the wall, so that they satisfy

$$f_{\pm}(r_{\pm})\dot{t}_{\pm}^2(\tau) - \frac{\dot{r}_{\pm}^2(\tau)}{f_{\pm}(r_{\pm})} = 1, \quad (2)$$

where a dot denotes derivative with respect to  $\tau$ . We take the radial coordinates so that the radius of the bubble is given by  $R = r_+ = r_-$  in both inner and outer coordinates.

The evolution of the bubble wall is described by the following equation [14, 20, 21] based on Israel's junction condition [22]

$$\beta_- - \beta_+ = 4\pi G\sigma R \equiv \Sigma R, \quad (3)$$

where  $\beta_+ \equiv f_+ \dot{t}_+ = \pm \sqrt{f_+ + \dot{R}^2}$  and  $\beta_- \equiv f_- \dot{t}_- = \pm \sqrt{f_- + \dot{R}^2}$ . From (3) we find the wall radius satisfies

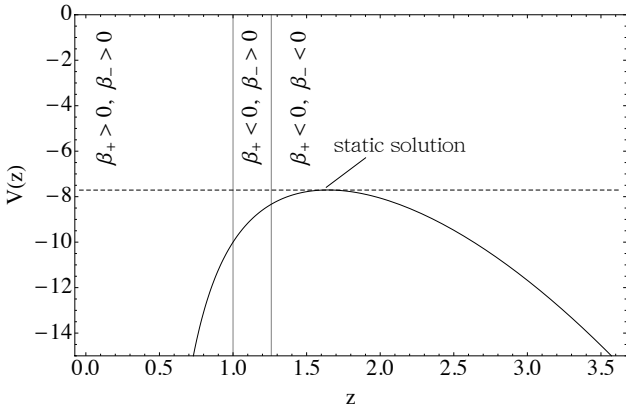


FIG. 2: Shape of the potential  $V(z)$  as a function of  $z$  with  $s = 0.9$ . We have taken  $\gamma = 1$  for illustrative purpose, although we actually expect  $\gamma \lesssim 10^{-3}$  for  $M_X \lesssim M_{\text{GUT}}$ . When  $E$  takes the value indicated by the dashed line, there is a static Euclidean solution of a bubble with a finite radius. For  $E$  smaller than this value, a growing or decaying Lorentzian bubble may be connected from an Euclidean solution.

the following equation similar to an energy conservation equation of a particle in a potential  $V(z)$ .

$$\left(\frac{dz}{d\tau'}\right)^2 + V(z) = E, \quad V(z) \equiv -\frac{1}{1-s} \frac{\gamma^2}{z} - \left(\frac{1-z^3}{z^2}\right)^2, \quad (4)$$

$$E \equiv -\frac{\gamma^2}{[2GM_+\chi(1-s)]^{\frac{2}{3}}}, \quad \chi \equiv (H^2 + \Sigma^2)^{\frac{1}{2}}, \quad \gamma \equiv \frac{2\Sigma}{\chi}. \quad (5)$$

Here dimensionless coordinate variables are defined by

$$\tau' \equiv \frac{\chi^2 \tau}{2\Sigma}, \quad z^3 \equiv \frac{\chi^2 R^3}{2GM_+(1-s)}, \quad \text{with } s \equiv \frac{M_-}{M_+}. \quad (6)$$

As is seen in Fig. 2, the potential  $V(z)$  has a concave shape with the maximum  $V(z_m) \equiv V_{\text{max}}$  given by

$$V_{\text{max}} = -3 \frac{z_m^6 - 1}{z_m^4}, \quad (7)$$

$$z_m^3 = \pm \left[ 2 + \left( \frac{1}{2} - \frac{\gamma^2}{4(1-s)} \right)^2 \right]^{\frac{1}{2}} - \left( \frac{1}{2} - \frac{\gamma^2}{4(1-s)} \right). \quad (8)$$

In Eq. (8), one must take a positive (negative) sign for  $s < 1$  ( $s > 1$ ), respectively. In this system, obviously, an Euclidean solution is possible if and only if  $E \leq V_{\text{max}}$ . Let us concentrate on the case  $E = V_{\text{max}}$  where there is an Euclidean solution of a static bubble, since  $E$  decreases in accordance with the decrease of the original black hole mass  $M_+$  due to the Hawking radiation. We calculate the Euclidean action of the instanton. This bubble is unstable in Lorentzian spacetime and may start expansion or contraction with the same probability after nucleation. We are of course interested in the case bubble wall expands after nucleation.

There are four relevant parameters in this system, namely,  $\chi$ ,  $\gamma$ ,  $M_+$ , and  $s$ . Among them,  $\chi$  and  $\gamma$  are determined by underlying high energy field theory. For the static bubble configuration, we find only the range  $s < 1$  is relevant, and from  $E = V(z_m)$  and  $V'(z_m) = 0$  we can express  $M_+$  and  $s$  as monotonic functions of  $v \equiv z_m^3$  as

$$s = \frac{v^2 + (1 - \frac{\gamma^2}{2})v - 2}{(v-1)(v+2)}, \quad (9)$$

$$M_+ = \frac{\gamma v}{3\sqrt{3}G\chi} \frac{v+2}{v+1} (v^2 - 1)^{-\frac{1}{2}}. \quad (10)$$

From the above analysis alone, one may think that  $s$  may take arbitrary small value down to  $s = 0$ . This is not the case, however, because the requirement that the time must proceed in the same direction in both inside and outside the wall, or in other words, that  $\beta_+$  and  $\beta_-$  must have the same sign imposes a nontrivial constraint on  $s$  [14]. For  $s < 1$ , in which we are interested, we find

$\beta_+ > 0$  for  $z > 1$  and  $\beta_+ < 0$  for  $z < 1$ . On the other hand, we find

$$\beta_- = \frac{1}{z^2 \sqrt{|E|}} - \left(1 - \frac{\gamma^2}{2}\right) \frac{z}{\sqrt{|E|}}. \quad (11)$$

Hence only for  $z > (1 - \gamma^2/2)^{-1/3} \equiv z_c$  we have  $\beta_- < 0$ . Thus in order for  $\beta_{\pm}$  have the same sign in the region  $z \geq z_m$ , where the nucleated bubble can expand, we must satisfy  $z_m > z_c$ , or  $v > (1 - \gamma^2/2)^{-1} > 1$ . Then we obtain the following bounds on  $M_+$  and  $s$  from (9) and (10).

$$M_+ < \frac{1 - \frac{\gamma^2}{3}}{2\sqrt{3}(1 - \frac{\gamma^2}{4})^{\frac{3}{2}}G\chi} \equiv M_c, \quad (12)$$

$$s > \frac{2}{3} \frac{1 - \frac{\gamma^2}{4}}{1 - \frac{\gamma^2}{2}} \equiv s_c. \quad (13)$$

We also find

$$\begin{aligned} M_- = sM_+ &= \frac{\gamma}{3\sqrt{3}G\chi} \frac{v^3 + (1 - \frac{\gamma^2}{2})v^2 - 2v}{(v^2 - 1)^{\frac{3}{2}}} \\ &< \frac{1}{3\sqrt{3}(1 - \frac{\gamma^2}{4})^{\frac{1}{2}}G\chi} \equiv M_{-c}. \end{aligned} \quad (14)$$

Thus physically relevant expanding bubble nucleation is possible only for  $\beta_+ < 0$  and  $\beta_- < 0$  satisfying the above bounds. It has been shown in [20] that in this case the trajectory of the bubble wall exists in region IV on the Penrose diagram (Fig. 4), that is, a wormhole-like configuration is created and the false vacuum bubble exists on the other side of the throat (Fig. 4-(c)).

Let us now calculate the Euclidean action corresponding to the static bubble configuration following Gregory, Moss, and Withers [14], according to whom the Euclidean action with a bubble  $I_o$  may be divided into the following components.

$$I_o = I_- + I_+ + I_W + I_B. \quad (15)$$

Here  $I_-$  and  $I_+$  denote contribution from inner and outer bulk, respectively, and  $I_W$  denotes that of the domain wall. Finally  $I_B$  represents contribution of conical deficits which was absent in Hiscock's analysis [13]. The result of explicit manipulation yields

$$I_o = -\frac{A_-}{4G} - \frac{A_+}{4G} + \int d\tau_E [(2R - 6GM_+) \dot{t}_{E+} - (2R - 6GM_-) \dot{t}_{E-}], \quad (16)$$

where  $A_-$  and  $A_+$  denote the area of the horizon of a Schwarzschild de Sitter black hole with mass  $M_-$  and that of a Schwarzschild black hole with mass  $M_+$ , respectively, and the suffix  $E$  indicates the Euclidean time.

The first and second terms, which are identical to the black hole entropy, are due to the conical deficits. It has been recently shown [23] that they are present even if we adopt Fischler-Morgan-Polchinski approach [24, 25] to calculate the transition rate using the WKB approximation, which justifies Gregory-Moss-Withers type expression of the bubble nucleation rate [14]

$$\Gamma \propto e^{-B} \quad \text{with} \quad B \equiv I_o - I_{Sch}. \quad (17)$$

Here  $I_{Sch}$  is the Euclidean action of the Schwarzschild black hole with mass  $M_+$ .

For the case of the static bubble, we find the last term on the right hand side of (16) vanishes [14]. As for the first term, the gravitational radius,  $r_{g-}$ , of the Schwarzschild de Sitter black hole with mass  $M_-$  is obtained by solving

$$f_-(r_-) = 1 - \frac{2GM_-}{r_-} - H^2 r_-^2 = 0. \quad (18)$$

We find

$$r_{g-} = \frac{2}{\sqrt{3}H} \cos \left[ \frac{\pi}{3} + \frac{1}{3} \arccos \left( 3\sqrt{3}GM_-H \right) \right] = \frac{2}{\sqrt{3}(1 - \frac{\gamma^2}{4})^{\frac{1}{2}}\chi} \cos \left\{ \frac{\pi}{3} + \frac{1}{3} \arccos \left[ \gamma \left( 1 - \frac{\gamma^2}{4} \right)^{\frac{1}{2}} \frac{v^3 + (1 - \frac{\gamma^2}{2})v^2 - 2v}{(v^2 - 1)^{\frac{3}{2}}} \right] \right\}. \quad (19)$$

The other positive solution of (18),  $r_{c-}$ , which would correspond to the cosmological event horizon in case of genuine Schwarzschild de Sitter space, is given by

$$r_{c-} = \frac{2}{\sqrt{3}H} \cos \left[ \frac{\pi}{3} - \frac{1}{3} \arccos \left( 3\sqrt{3}GM_-H \right) \right]. \quad (20)$$

We find it is larger than the bubble radius,

$$R = \frac{\gamma v}{\sqrt{3}\chi(v^2 - 1)^{\frac{1}{2}}}. \quad (21)$$

Therefore, there is no cosmological horizon at this stage. This is why we have only black hole entropy terms in (16) unlike [14].

It is interesting to note that in the limit  $z_m = z_c$  ( $M_+ =$

$M_c$ ), we find

$$r_{g-} = r_{c-} = R = \frac{1}{\sqrt{3}(1 - \frac{\gamma^2}{4})^{\frac{1}{2}}\chi} = \frac{1 - \frac{\gamma^2}{4}}{1 - \frac{\gamma^2}{3}} r_{g+} > r_{g+}, \quad (22)$$

yielding  $r_{g-} > r_{g+}$ . Once we take  $z_m > z_c$  ( $M_+ < M_c$ ), the proper hierarchy  $r_{g-} < R < r_{c-}$  is maintained but the inequality  $r_{g-} > r_{g+}$  still holds (solid lines in Fig. 3).

As a result we find

$$B = I_o - I_{Sch} = -\frac{A_-}{4G} \quad (23)$$

takes a negative value because  $M_-0$  takes a finite value due to the constraint (13). It may suggest spontaneous nucleation of a bubble soon after the mass of the original black hole falls below the critical value  $M_c$ . This result may better be interpreted from thermodynamic point of view [26, 27]. As the terms corresponding to the energy [28] are absent in both  $I_o$  and  $I_{Sch}$ , the transition may be determined by the increase of the entropy by  $A_-/4G$ .

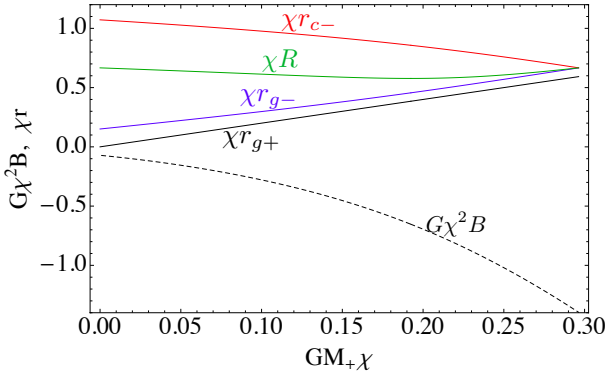


FIG. 3: The horizons  $r_{c-}$  and  $r_{g-}$  of the Schwarzschild de Sitter black hole, the bubble radius  $R$ , and the horizon of the Schwarzschild black hole  $r_{g+}$  are shown in unit of  $\chi^{-1}$  as a function of  $M_+$ . Also drawn is  $G\chi^2 B$ . The inequality  $r_{g-} > r_{g+}$  and the negativity of  $B$  hold in the range of  $0 < M_+ \leq M_c$ .

Then we can sketch the following scenario of cosmic evolution. Typical astrophysical black holes with mass  $\sim 10M_\odot$  will evaporate in  $\sim 10^{67}$  years from now. As its mass falls below the critical value  $M_c$ , a false vacuum bubble is spontaneously nucleated with the radius (21). It is unstable in Lorentzian spacetime and so the bubble would start expanding with the probability 1/2, and then a wormhole-like configuration is realized. The space on the other side of the throat starts inflation to create an exponentially large domain causally disconnected from our patch of the universe. If inflation is appropriately terminated followed by reheating, another big bang universe will result there. For this purpose the old inflation model

[4, 5] with thin wall bubble nucleation does not work, but we may make use of the results of open inflation models there [29–31] which can also realize an effectively flat universe. Throughout these processes, the outer geometry remain Schwarzschild space with the mass parameter  $M_+$ , so those who live there do not realize a black hole in their universe has created a child universe. The above result may also suggest that our Universe may have been created from a black hole in a previous generation in the cosmos.

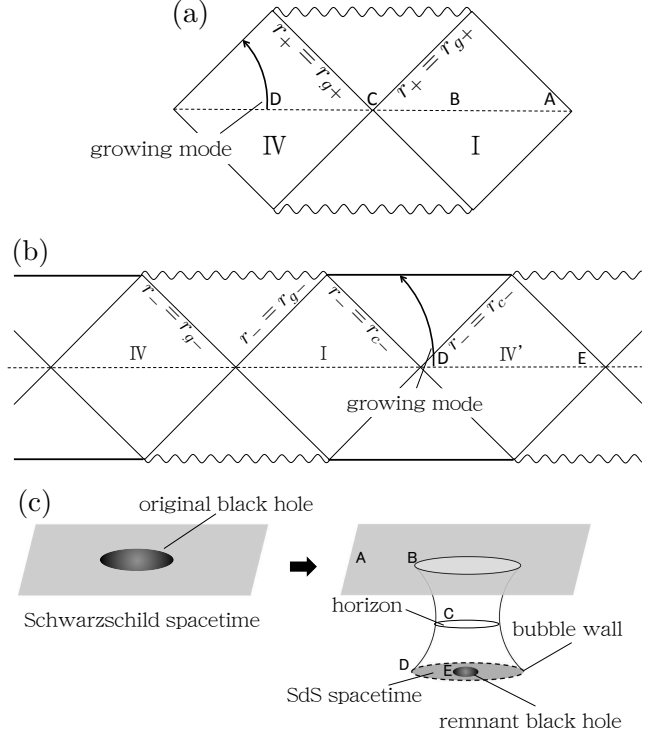


FIG. 4: The trajectories of a bubble wall on Penrose diagrams and a schematic figure of the structure of a false vacuum bubble induced by a phase transition. Figure (a) shows the spacetime outside the wall and Figure (b) that inside the wall. Figure (c) is a schematic figure of a false vacuum bubble derived from the solution of Israel junction. The false vacuum bubble is on the other side of the throat constituting a child universe. The throat has the horizon with the radius  $r = r_{g+}$  at the moment of the nucleation of a child universe.

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